

The role of spectral curvature mapping in characterizing subsurface strain distributions

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Abstract: The curvature of structured geological surfaces can be used to assess the degree of strain they have undergone. In many hydrocarbon reservoirs, this strain is expressed as brittle fracturing that may significantly impact reservoir performance. Here we describe the development of an algorithm for measuring the curvature of gridded surfaces derived from seismic data. For any grid node, the algorithm calculates the magnitude and orientations of the two principal curvatures, K_1 and K_2 , from which other curvature measurements can be derived, such as Gaussian curvature and summed absolute curvature ($K_1 + K_2$). The algorithm has also been used to generate plots of summed absolute curvature as a function of grid node separation (k versus λ). These 'spectral' or $k\lambda$ plots can be generated for each grid node and allow the definition of short-wavelength, high-amplitude noise cut-off lengths. They also deliver intermediate wavelength features such as fault drag or buckle folding and the identification of long-wavelength (basin-scale) curvatures. Portions of these data can be collapsed into single values by calculating the integral of the $k\lambda$ curve. Further filters designed to screen the effects of background tectonic, or non-tectonic, curvatures can be applied to the $k\lambda$ integral.

This algorithm has been tested using data from several North Sea chalk fields. A range of alternative types of curvature and curvature spectra are compared with other approaches to curvature calculation and other factors relevant to the calibration of such techniques in terms of the distribution of brittle fractures in sedimentary rocks.

The $k\lambda$ integral provides a relatively simple approach to calculating the degree of multi-wavelength strain present at a particular grid node. Freeing algorithms from the restriction of the 'arbitrarily' selected minimum grid node spacing is a key step towards calibrating measured curvature against strain mechanisms. However, care must be taken to separate intrinsic and tectonic curvatures when generating and interpreting $k\lambda$ plots and their integrals.

The presence of small-scale faults and fractures in a hydrocarbon reservoir can have a significant impact on the permeability characteristics of the reservoir. Small-scale fractures can be defined as those not explicitly defined by 'remote' mapping methods such as 3D seismic data or aerial photography. It has long been recognized that fold-related curvature may result in fracturing important for oilfield permeability. Murray (1968) related the radius of curvature of a competent unit to fracture porosity for an oilfield in North Dakota. Stearns (1964) and Stearns & Friedman (1972) provided models of fracture orientations and distributions for non-cylindrical folds which have proved an enduring reference point for fold-related fractures in hydrocarbon reservoirs (Cooke-Yarborough 1994). A number of studies have related the degree of curvature of a bed or sequence of beds and the fracture densities mapped within the bed. Relatively recent examples include Narr (1991) who related increased fracture density to the location of a

doubly plunging fold hinge in the Point Arguello oilfield, California and Ericsson *et al.* (1998) who defined a strong relationship between fracture density and curvature for the Fateh Field, offshore Oman.

The curvature of a cylindrical fold structure can be defined as the reciprocal of the radius of curvature (fold radius) measured in a plane perpendicular to the fold axis (the profile plane, see Fig. 1a). In a more complex fold structure, such as those commonly mapped from 3-D seismic surveys, the folding may not be adequately described by one component of curvature. Lisle & Robinson (1995) detailed a more sophisticated method for measuring the curvature of geological surfaces, which is represented by two orthogonal principal curvatures, the maximum (K_1) and the minimum (K_2) principal curvatures. The product of these two curvatures gives the Gaussian curvature, which is a measure of the amount of double curvature of a fold structure (see Fig. 1b).

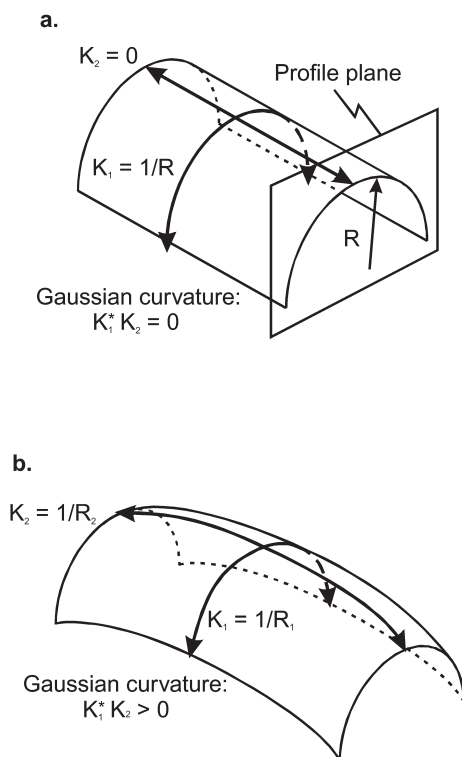


Fig. 1. (a) Model of a cylindrical fold with the principal curvatures and the profile plane shown. The curvature is measured within the profile plane.; (b) Model of a doubly curved fold surface. The product of the two curvatures is not equal to zero and is termed the Gaussian curvature.

There are several methods for measuring curvature (Stewart & Podolski 1998); mapped surface dip or azimuth change (Ericsson *et al.* 1998; Steen *et al.* 1998); position with respect to the crest of an anticline or pericline (McQuillan 1973; Agarwal *et al.* 1997; Hanks *et al.* 1997; Ericsson *et al.* 1998) or Gaussian curvature (Lisle 1994; Agarwal *et al.* 1997). Whilst these methods are sufficient in many cases, they have little or no potential to accurately measure the principal curvatures of a surface in most situations. The purpose of this paper is to outline a practical implementation of the methods for measuring curvature outlined in Lisle & Robinson (1995) coupled with a method for representing and analysing curvature variations across several spatial wavelengths. The algorithm used is described and several examples of spectral curvature estimates are presented from North

Sea oilfield reservoir surfaces. These examples are compared and discussed with respect to models of curvature versus strain and other factors influencing the development of fractures in deformed strata.

Algorithm for surface curvature

The principles of surface curvature outlined in Calladine (1986), Nutbourne & Martin (1988) and Lisle & Robinson (1995) allow the description of various parameters of curvature on a surface. These papers define the characteristics of the space curve and surface curve reference frames that allow the magnitudes and orientations of the principal curvatures to be calculated. It is beyond the scope of this paper to reiterate the principles and methodologies outlined in the above papers. However, the algorithm used to calculate the curvature parameters discussed in this paper draws heavily on these principles. The reader is referred to those works to gain a more detailed knowledge of curvature analysis.

A computer algorithm has been developed based on the methodology outlined in Lisle & Robinson (1995). The algorithm calculates the principal curvature magnitudes and orientations from an orthogonal grid of points on a surface, such as those obtained from 3D seismic mapping packages. Figure 2 shows a summary of the process for calculating the principal curvatures for a given grid node. The input and output formats of the algorithm are grids of XYZ data with orthogonal X and Y directions. To provide meaningful curvature values, the XYZ values must all be in the same units; in the real examples used here the units are metres. Processing time is reduced if the grid boundaries are parallel to the grid directions, but the algorithm can handle input and output grids with irregular boundaries. The preferred grid geometry is for points with equal spacing in the X and Y directions (square grids). Grids with unequal X and Y direction spacings (rectangular grids) will result in bias of the curvature measurements. This is shown in Figure 3a where three curvature estimates are clustered spatially on a highly anisotropic rectangular grid. The algorithm also makes corrections for diagonal direction curvature calculations to compensate for the extra length (equal to a factor of root 2 in orthogonal, equispaced grids). This can only be accomplished at grid offsets greater than the minimum grid spacing. Figure 3b shows the general methodology for calculating the curvature variables using grid offsets of $2s$ and $8s$ (where s is the grid spacing). The following variables are output by the algorithm: K_1 , K_2 , Absolute K_1 +Absolute K_2 , Gaussian

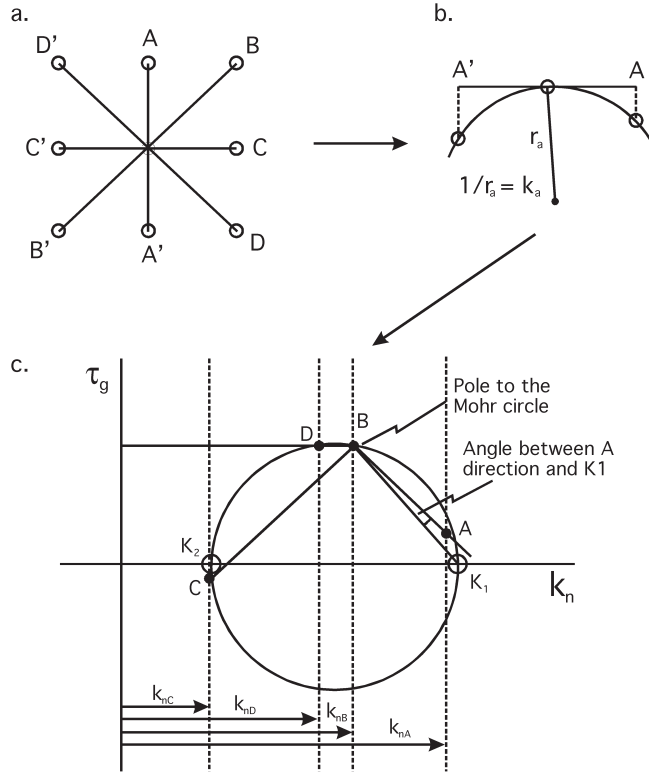


Fig. 2. Features of the algorithm for calculating curvature from gridded data points. (a) Plan view of arrangement of a node with its eight nearest neighbours. Curvature is measured on four vertical planes through the central node and opposite pairs. The planes are labelled A, B, C and D. (b) Section view of three nodes. The curvature is calculated by fitting an arc of a circle through the three points, finding the coordinates of the centre and calculating the radius of the circle. Curvature is the reciprocal of the radius and is calculated for each set of A–A', B–B', C–C' and D–D'. The curvatures are then plotted on a Mohr circle diagram. (c) The Mohr circle construction allows plotting of the data points in normal curvature (k_n) versus surface torsion (τ_g) space without prior knowledge of the position of the k_n axis. The procedure for calculating normal curvatures and surface torsion is described in Lisle & Robinson (1995). The principal curvatures lie in directions of no surface torsion. A least squares best fit circle is fitted through the four points and the centre used to position the k_n axis. The principal curvatures K_1 and K_2 are calculated from the intersection of the circle with the k_n axis.

curvature, theta and the root mean square variation of the best fit circle to the data on the Mohr circle construction. Theta is the angle between the K_1 direction and the grid Y direction and ranges from 0° to 180° clockwise from the Y direction. Figure 3b also illustrates the reduction in length of the diagonal section planes at larger grid offsets. A key feature of the algorithm is the ability to calculate the curvature variables across the grid at consecutive grid offsets or at multiples of the consecutive grid offsets. Therefore, for each grid node it is possible to calculate the six output variables at many different sampling offsets. This allows the construction of the spectral curvature plots, which are discussed below.

Absolute $K_1 + \text{Absolute } K_2$ (referred to as $K_1 + K_2$) is used in this study as it is a good indicator of the total curvature affecting a point (Stewart & Podolski 1998). Considerable strain may be associated with cylindrical or near-cylindrical fold structures; however, Gaussian curvature values for these structures will show low or zero values because K_2 will be low or zero. It is unlikely that any features will be exactly cylindrical, but the very low values of K_2 will correspondingly reduce the Gaussian curvature values. The absolute values are used in the $K_1 + K_2$ measure because just using the sum of the principal curvatures can lead to low or zero values if one curvature is positive and the other

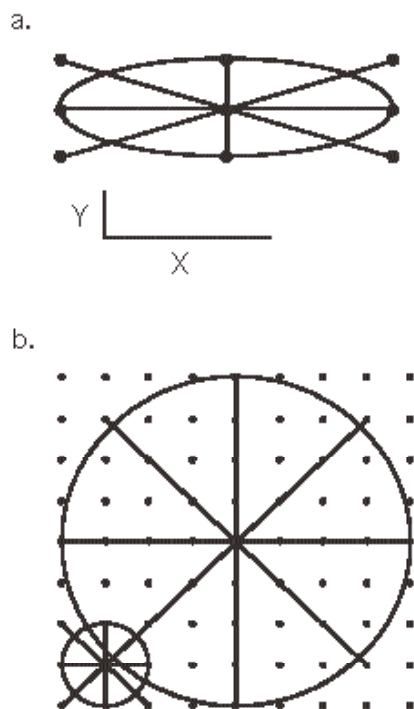


Fig. 3. (a) Plan view of an example of a highly anisotropic grid with X spacing approximately three times the Y spacing. It can be seen that the curvature is undersampled in the X direction compared with the Y direction. (b) Curvature can be calculated at a range of grid offsets. The small circle shows the data points used for an offset of one grid spacing either side of the chosen node. The larger circle shows the arrangement of points for an offset of four grid spacings. Note that the diagonal measurements on the larger circle are at an offset of three to compensate for the extra distance between the nodes in the diagonal direction. The optimum diagonal offset can be calculated for all offsets greater than one.

negative, as in saddle structures. The curvature sign convention used in this paper is positive for convex-up curvatures. In this paper, K_1 , K_2 and $K_1 + K_2$ are expressed in units of m^{-1} and Gaussian curvature is expressed in units of m^{-2} .

Relationship of curvature to geological structures

A variety of different structural features can result in changes of curvature of a geological surface such as a bed boundary. Obvious examples are buckle folds, fault related folds, basement-controlled structures (e.g. inversion anticlines in post-rift cover) and salt domes.

Buckle folds

The two mechanisms of folding relevant to buckle folds are tangential longitudinal strain and flexural slip (Ramsay & Huber 1987; Price & Cosgrove 1990). If the curvature of a surface can be shown or inferred to have resulted from tectonic activity, then an additional problem is how to relate the curvature to brittle strain within a unit or group of units. The strains in a cylindrical fold can be defined by the following relationship (from Price & Cosgrove 1990)

$$\epsilon = kt/2 \quad (1)$$

where ϵ is the fibre strain in the outer arc of the fold (assuming tangential longitudinal strain), k is maximum curvature and t the thickness of the folded unit. However, it is generally not clear where the neutral surface of a lithological unit lies with respect to the gridded surface. As seismic reflectors are produced at acoustic impedance contrasts, bed boundary reflectors will dominate them. If the top of a competent lithomechanical unit has been imaged, then the extensional strains will be at a maximum over areas of positive curvature. Core data will help to constrain the lithostratigraphy and if combined with field analogue data can provide an estimate of the mechanical stratigraphy. In addition, several outcrop studies of large-scale folds dominated by the flexural slip folding mechanism indicate that fracturing distributions can be unrelated to curvature or even concentrated in the relatively planar fold limbs rather than the more curved fold hinges (Jamison 1997; Hank *et al.* 1997; Couples *et al.* 1998). Clearly an assessment of the appropriate folding mechanism is important when trying to relate mapped surface curvature to strain.

Fault-related curvature

Extensional fault offsets of a bedding surface produce a number of issues with respect to curvature mapping. Faults with large throws (several hundred metres plus) in a surface with an area of a few tens of square kilometres will dominate the curvature spectra, especially at points adjacent to the fault. However, this effect is primarily a result of the offset of the beds and not necessarily related to the curvature of the bedding surface itself. In these situations it is recommended that the curvature mapping is subdivided into subareas defined by these large-scale faults (Stewart & Podolski 1998). This will allow the mapping of curvature on rollover anticlines (Gibbs 1984), fault drag folding or microfaulting immediately adjacent to the fault (Steen *et al.* 1998) without interference from the large-scale fault offset.

Basement-controlled features in cover rocks

A number of features may form in sedimentary rocks deposited unconformably over a pre-existing structure in the underlying basement rocks. On a large scale, these features could be monoclinical flexures of the cover sequence that either formed over an existing topographic feature or were formed by reactivation of the basement structure. Another common manifestation of basement reactivation is the propagation of faults and fractures into the cover sequence. Such features have been described from many oilfields such as the Asmari Formation reservoirs, SW Iran (McQuillan 1985). Clearly, it is important to separate flexures produced from the drape of cover over topography from flexures produced by reactivation of the basement features. The former are unlikely to contain faults or fractures systematically related to the fold geometry. To differentiate between these two mechanisms some form of geological history needs to be established for the reservoir. This could be built up from identifying the facies types and structures from well data and assessing their relative timings. An input to this could be basin development modelling incorporating strain restoration techniques.

Salt-cored features

Salt-cored features are another common mechanism for forming structural closures with the potential for trapping hydrocarbons. Salt domes are rarely perfectly symmetrical and often form periclinal features (Jackson *et al.* 1994). Fault structures associated with salt domes may be radial such as those found on the Kyle Field, North Sea, or they may be aligned to one or both of the periclinal axes. Withjack & Scheiner (1982) have shown from experimental modelling that extensional faults form radial patterns on the flanks of circular and periclinal domes in the absence of an applied regional strain. With an applied extensional strain, the extensional faults become aligned perpendicular to the extension direction. Clearly the fault geometries on such structures are not determinable from the contemporary curvature alone. If the faulting on such a structure cannot be directly resolved from remote mapping methods (e.g. 3D seismic), then an appreciation of the strain history of a structure may be required in conjunction with the curvature analysis of a particular surface.

Spectral curvature analysis

Variable versus offset ($v-\lambda$) plots

The various types of data output by the algorithm outlined above can be plotted on a variable (v)

versus grid offset (λ) plot (cf. Stewart & Podolski 1998). These plots show the variations of curvature with increasing offset from a single grid node and are a novel way of analysing the spectrum of curvature data measured at a single point. They represent a complementary approach to using multiple colour-shaded or contour maps of curvature at different grid offsets. The sections below describe the main features of the $v-\lambda$ plots for the Top Ekofisk chalk surface of the Fife Field, North Sea. $K_1+K_2-\lambda$ plots from the Top Tor surface of the Valhall Field are also discussed. The surfaces from both fields were depth-converted prior to calculating the $v-\lambda$ spectra.

Characteristics of spectral curvature plots

The curvature parameters plotted are K_1 , K_2 , K_1+K_2 and Gaussian curvature. All the plots show a general trend of extreme values at smaller offsets that change in a non-linear way to less extreme values at larger offsets. Figure 4 shows the main characteristics of typical K_1+K_2 spectral plots. Figure 4a shows the effects on K_1+K_2 of superimposing random noise on a plane. This was achieved by adding a random number to each grid node in the range $\pm 0.1\%$ of the grid spacing. This plot was included to allow a comparison to the average of the K_1+K_2 spectra of the Top Ekofisk surface from the Fife Field shown in Figure 4b. The plots have very similar shapes and highlight the importance of short-wavelength features on the characteristics of the curvature spectra at sampling wavelengths greater than the noise wavelength. The similarity between the two plots in Figure 4 is due to the effect of aliasing from a sampling wavelength that is greater than the wavelength being measured. The aliased wavelengths are larger than the actual wavelengths causing the slope on the spectral plots to decrease over an interval rather than at a single point. To effectively sample a wavelength without aliasing, the sampling wavelength must be less than or equal to half the smallest wavelength of interest. This minimum wavelength is termed the Nyquist wavelength and in the practical situations described here, is determined by the minimum grid spacing (Stewart & Podolski 1998).

From the spectral plots of random noise on a planar surface shown in Figure 4a it can be seen that artefacts can occur at intermediate offsets that are not related to any genuine curvatures. Therefore, care must be taken when interpreting any specific features from individual spectral plots. However, a characteristic feature of the spectra from the noise superimposed on the

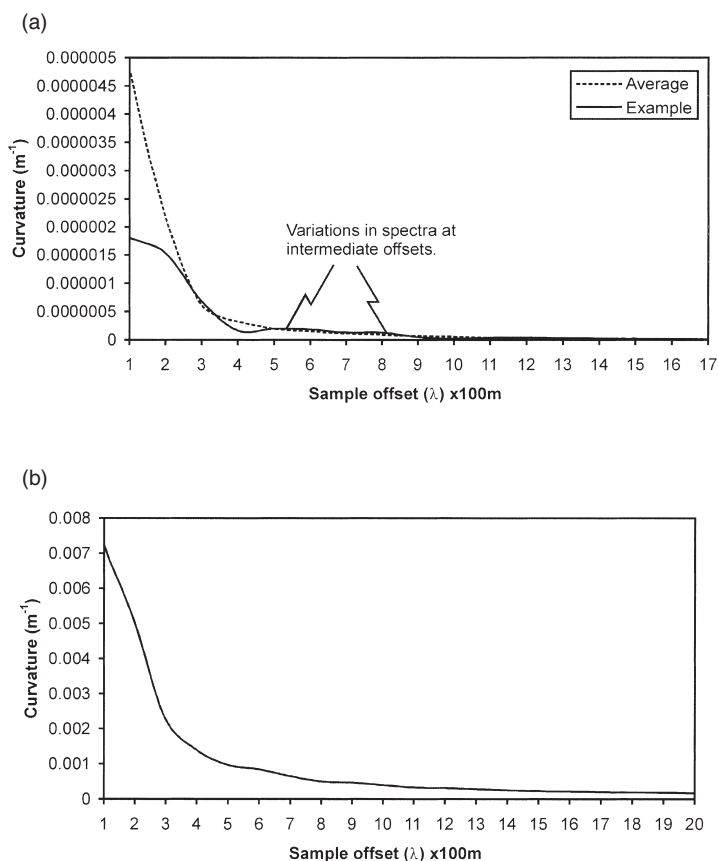


Fig. 4. (a) Average spectra and an example from a grid of points on a plane with random noise added to each grid node at $\pm 0.1\%$ of the grid spacing. Note the similarity of the spectra to (b) which is the average $K_1 + K_2$ spectrum for the Fife Field Top Ekofisk surface.

planar surface is that the slope of the curve at smaller offsets is much steeper than that for the average of spectra of the Fife $K_1 + K_2$ dataset. This can be quantified from the fitting of power law trend lines that appear to give the best fit to the average data from both the noise on the plane plot and data from the North Sea fields. Two North Sea fields were used, the Fife Field and the Valhall Field, both situated in the East Central Graben. The locations of the spectra discussed below are shown on Figure 5 for the Fife Field and Figure 6 for the Valhall Field.

On both plots shown in Figure 4, the trend line R^2 values are greater than 0.99. The power law exponent for the trendline fitted to the average $K_1 + K_2$ spectra of noise superimposed on a plane is -2.0266 . The trendline power law exponents for the average spectra of the real datasets are -1.3427 for the Fife $K_1 + K_2$ data and -1.1355 for the Valhall $K_1 + K_2$ data. The

averages of the spectra from both the Fife and Valhall datasets were taken from spectra calculated on subgrids across the surfaces. A total of 132 spectra were used for the Fife average and 106 spectra for the Valhall average. The difference in power law exponent between the plane with noise and the real datasets may allow differentiation between-noise dominated data and natural curvature features. However, a detailed examination of the power law (fractal) characteristics of surfaces and their relationship to spectral curvature plots is beyond the scope of this work.

Spectral curvature plots from North Sea chalk fields

Figure 5 shows the topography of the Top Ekofisk surface from the Fife Field used in this study. The data are in a grid with 50 m spacing

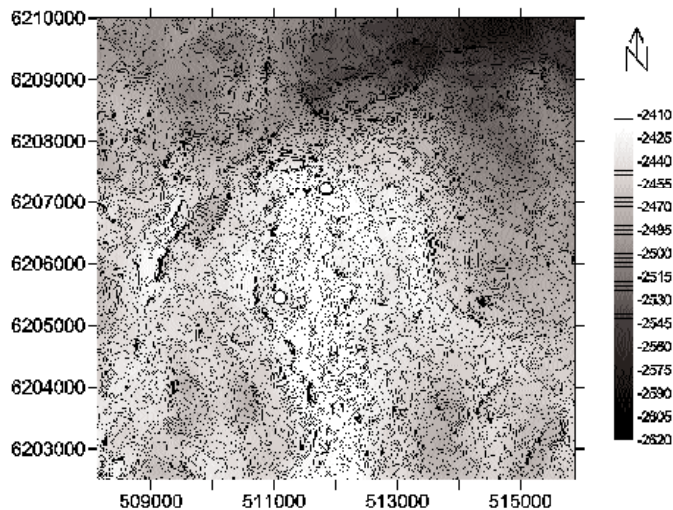


Fig. 5. Contour map of the Top Ekofisk surface for the Fife Field, East Central Graben. The surface has been depth converted and regridded to 50 m. No smoothing has been applied to the data. All values in metres.

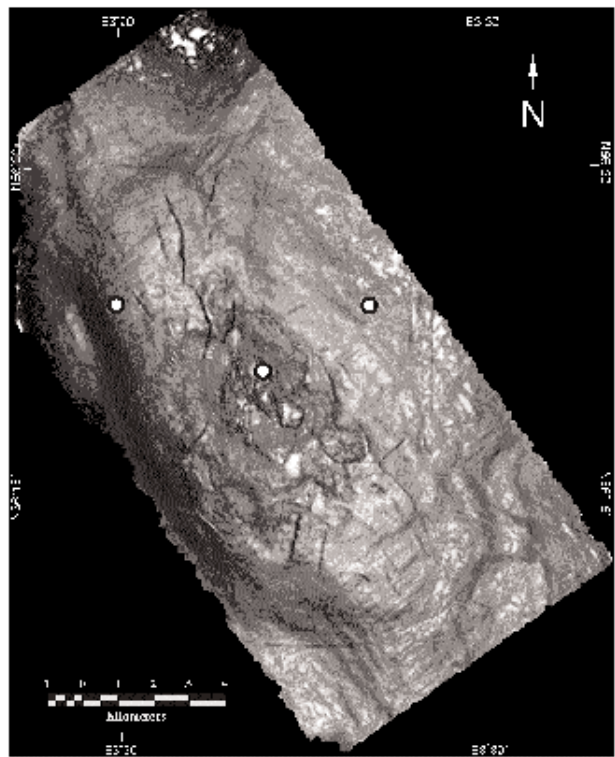


Fig. 6. Shaded relief map of the Top Tor surface for the Valhall Field, East Central Graben. The surface has been depth converted and no smoothing was applied to the data. The grid spacing is 50 m.

that was regridded from a 25 m spacing. The regridding was performed to reduce the dataset to a manageable size, although this will result in aliasing of the shorter (*c.* 50 m) wavelength features. It can be seen that in the centre of the field is a broad, flat-topped dome. The northerly closure is steeper than the east and west closures and represents topography of the chalk over a basement feature. This is probably a fault reactivated in the Late Cretaceous to Palaeogene (Gowers *et al.* 1993). There is also part of a smaller, steep-sided closure visible west of the main dome (E509200, N6206000). The north-south to NNW-SSE trending feature on the west side of the dome may represent reactivated faulting affecting the overlying chalk surfaces. It can also be seen that a number of 500 m to 1000 m scale domes and basins occur on the surface which will affect any curvature measurements. It is possible to smooth a surface prior to calculating the curvature to remove the effects of noise (Stewart & Podolski 1998). However, gentle smoothing is unlikely to remove all the noise, and aggressive smoothing is likely to remove some or all of the natural short-wavelength features. For this reason, it was decided to use an unsmoothed surface in this study.

The main characteristics of the $K_1 + K_2$ spectra from the Top Ekofisk surface allow a description of the spectra in terms of the combination of two main components. The first is a short-wavelength, high-amplitude curvature that may represent random 'noise' associated with the acquisition and processing of the seismic data (Stewart & Podolski 1998; Brown 1991). The second component represents long-wavelength,

low-amplitude curvature that probably represents the overall 'background' curvature of the basin. It must be noted that not all of the short-wavelength, high-amplitude curvature will be noise: some will represent the effects of real features such as parasitic folds or faults.

Superimposed on the general negative slope of the plots can be a variety of local changes in the rate of decrease of slope or even local increases in slope. These variations are discussed below where a variety of $k\lambda$ plots with departures from the general form are shown. For the unsmoothed Top Ekofisk surface of the Fife Field data, these departures can occur at any sampling wavelength but are most common at intermediate wavelengths of about 400 m to 1300 m.

Fife Field K_1 and K_2 plots

Figure 7 shows typical spectra for the K_1 and K_2 variations at the crest of the main dome on the Top Ekofisk surface and the north-south trending ridge on the west side of the main dome (see Fig. 5 for locations). These spectra are highly variable at smaller offsets and become less variable at larger offsets; the values vary from positive to negative for both K_1 and K_2 but generally seem to converge on very low (<0.0003) positive curvatures at offsets greater than about 1700 m. The exception is the west flank K_1 curve that displays negative values at all sample offsets. There also appear to be changes in the slopes of all the spectra in Figure 7 at sample offsets of 700–900 m. This may reflect a characteristic wavelength of the Fife Field although it is not

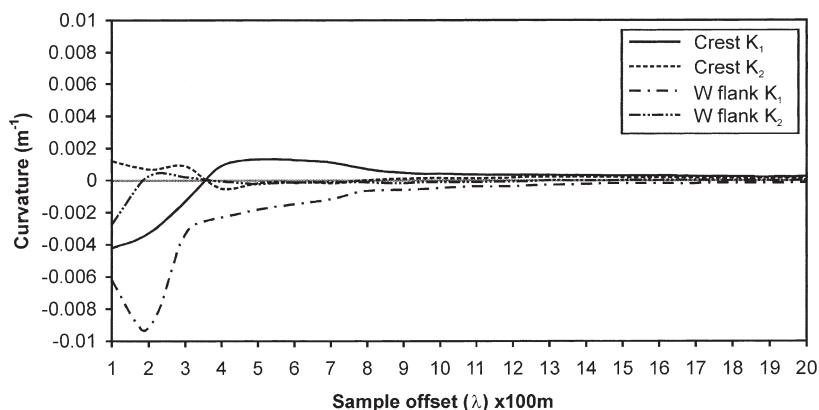


Fig. 7. K_1 and K_2 versus sample offset spectra from two grid nodes on the crest and the west flank of the Fife Field Top Ekofisk surface.

clear if the origins of features at this wavelength are tectonic or sedimentary.

Fife Field K_1+K_2 plots

Figure 8 shows the basic features of a typical K_1+K_2 versus λ plot. It can be seen that in common with the $k\lambda$ plots for K_1 and K_2 , higher curvature values are measured at smaller offsets that fall off to a background level of lower curvatures at higher offsets.

The spectra taken from the crest show relatively low values at sample offsets of 100 to 400 m when compared to the spectra from the west flank of the dome. However, at sample offsets of 700 m to 1200 m the curves converge

and at sample offsets of 1200 m to 2000 m the spectra from the crest show higher values. These characteristics are to be expected for these locations as the crest of the dome shows little topography on the 100 m to 500 m scale; conversely the west flank of the dome is dominated by the north-south trending ridge that produces the high values at sample offsets of 100m to 400m.

Valhall Field K_1+K_2 plots

Figure 6 shows the Top Tor surface from the Valhall Field, East Central Graben. The locations of the spectra shown in Figure 9 are also marked. The origins of this field are similar to

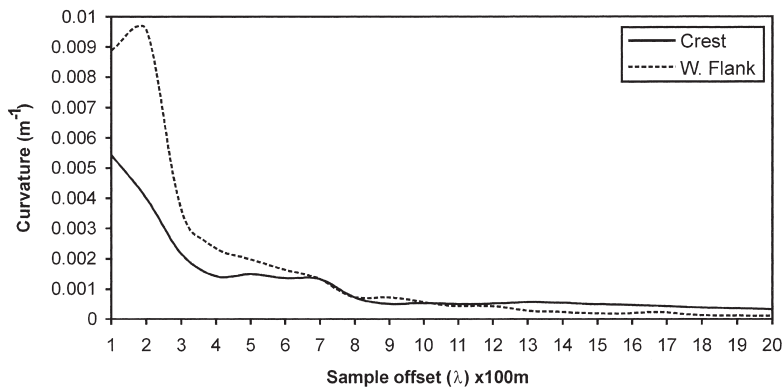


Fig. 8. Absolute K_1 +Absolute K_2 (K_1+K_2) versus sample offset spectra from two grid nodes on the crest and the west flank of the Fife Field Top Ekofisk surface. Note the localized increase in curvature at 400 m sample offset for the crestal node.

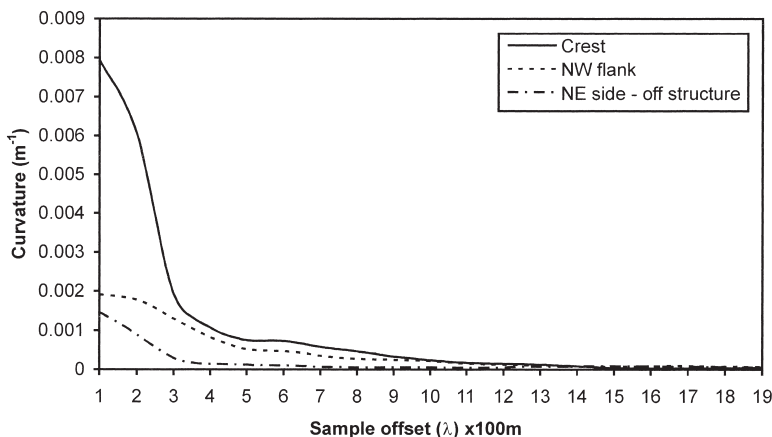


Fig. 9. Absolute K_1 +Absolute K_2 (K_1+K_2) versus grid offset spectra from three grid nodes on the Valhall Field Top Tor surface. The nodes are situated on the crest of the main pericline, the NW flank of the pericline and the NE side of the dataset (off structure).

those of the Fife Field in that the main structure formed during late Cretaceous to Palaeogene inversion of the Feda Graben (Gowers *et al.* 1993). It can be seen that there are numerous faults concentrated around the crestal part of the main NW–SE trending pericline. The grid spacing for this dataset is also 50 m but the overall field size is much larger than Fife. The spectra show different characteristics, which appear to be related to their locations on the Top Tor surface. The spectrum from the crest shows very rapid decay from high values (0.006–0.008) at sample offsets of 200 m to 300 m. The curve decreases to low values (<0.0002) at offsets greater than 1100 m. This may be related to the fact that the crestal part of the pericline is relatively flat with the topography dominated by the faulting. The spectrum from the NW flank of the main pericline shows much lower values at sample offsets up to 300 m. This is expected as there is little or no faulting in this part of the structure. At sample offsets greater than 300 m, the NW flank spectrum is similar in shape to the crestal spectrum. This indicates that both spectra are controlled by the main pericline structure at sample offsets greater than 300 m. The spectrum from the NE side of the dataset shows consistently lower values than the other two spectra up to sample offsets of 1400 m. This can be related to its location on the relatively flat NE flank of the structure with little or no faulting and minimal influence from the main pericline structure.

The general shapes of the spectra are similar to those observed for the Fife Field but the slopes of the Valhall curves decay to lower values

(<0.0002) at sample offsets of 1000 m to 1100 m for the spectra associated with the pericline. For the spectra associated with the flank this value is reached at a sample offset of 300 m. The $K_1 + K_2$ spectra for the Fife Field tend to decay to these values at sample offsets of 1300 m to 1400 m. This difference can be related to the topography of the two surfaces. The Valhall Top Tor surface appears to be a relatively simple, large-scale pericline with 1–3 km long faults clustered around the crest. Therefore, the faulting and the large-scale pericline influence the curvature which occurs at two different scales. However, the Fife Top Ekofisk surface shows a greater degree of variability in topography at a wider range of scales than the Valhall Field. This spread of scales for the features in the Fife dataset has probably contributed to 'holding up' the decay of the short-wavelength curvature from short to long sample offsets.

Fife Field Gaussian curvature plots

Figure 10 shows the variations in Gaussian curvature with sample offset. The features of these plots are similar to those for the K_1 and K_2 plots. However, in general the variability at intermediate offsets (400 m to 1300 m) is less than for either the K_1 , K_2 or the $K_1 + K_2$ plots. All the curvatures measured at the scale of the Fife Field are less than unity. Therefore, when the product of K_1 and K_2 is calculated, the magnitude of the result is often considerably less than either of the two components. Consequently, Gaussian curvature from the Fife Field is more usefully plotted on maps to show the distribution

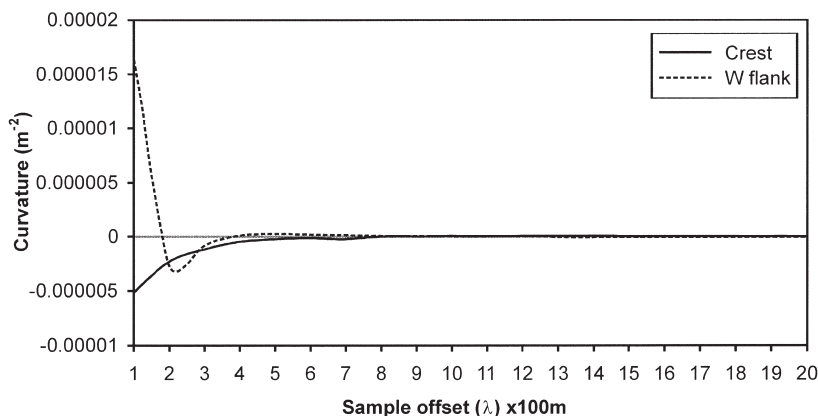


Fig. 10. Gaussian curvature versus grid offset spectra from two grid nodes on the crest and the west flank of the Fife Field Top Ekofisk surface. The curvature values for these spectra fall to very low values at sample offsets of approximately 400 m to 500 m.

of non-cylindrical curvature across the field. Alternatively, showing only the curves above a certain cut-off value may better represent these plots. This would allow an examination of the more subtle variations.

Fife Field integrals of $K_1 + K_2$ plots

The spectral plots for the variables discussed above provide a useful insight into the wavelengths and magnitudes of curvatures affecting each grid node. However, for each grid node, six different spectral plots can be generated. In large gridded datasets it is not uncommon to have in excess of 500 nodes in each grid direction resulting in 250 000 nodes or more. Clearly, the time required for the analysis of individual plots becomes prohibitive for anything but the smallest subareas of a dataset. A method is required to extract the relevant information from one or more of these plots and present it as a map of values. In this study, it was decided to use the integral of the $K_1 + K_2$ plots to collapse the multi-wavelength information into a single value for each node. The $K_1 + K_2$ plots are amenable to this analysis because they are always plotted in the same Cartesian quadrant and an integral will allow a qualitative comparison of collapsed curvatures when plotted on a map. Using the integrals of the curve also allows the removal of components of curvature associated with noise at shorter wavelengths and basin scale curvature at longer wavelengths.

The integral is calculated by using the trapezium rule, which is adequate for data of this type. Figure 11 shows the main features of the

method for calculating the integral and removing the noise- and basin-scale-related integrals. The noise-related curvatures are removed by applying a predetermined lower cut-off for the data, in this case 300 m, prior to calculating the integral. It is realized that this is a crude approach and because of the gradual decay of short-wavelength features over a range of wavelengths, some short-wavelength-related curvature will remain. The basin-scale curvatures are identified from the minimum measured curvature at a particular node with the assumption being that the smallest curvatures occur at the largest offsets. Removal of this basin-scale curvature has the effect of subtracting a constant value from the integral. The presence of uncharacteristically low curvatures at small or intermediate offsets will reduce the effectiveness of this approach. However, a visual inspection of the spectra from the Fife Field Top Ekofisk dataset revealed, for this dataset, that this is unlikely to occur. It may also be possible to filter the noise and basin-scale curvatures by integrating between the raw spectra and a normalized power law curve representing the contribution of noise- and basin-scale curvatures in the dataset. The input parameters for such a 'filter spectrum' may be determined by examining selections of spectra or the average of the spectra for the surface being considered.

After removal of the noise- and basin-scale-related integrals, the remaining area may give some measure of the different wavelengths of tectonically induced curvatures affecting the surface at any one point. The advantage of using this approach is that it is simple to calculate and

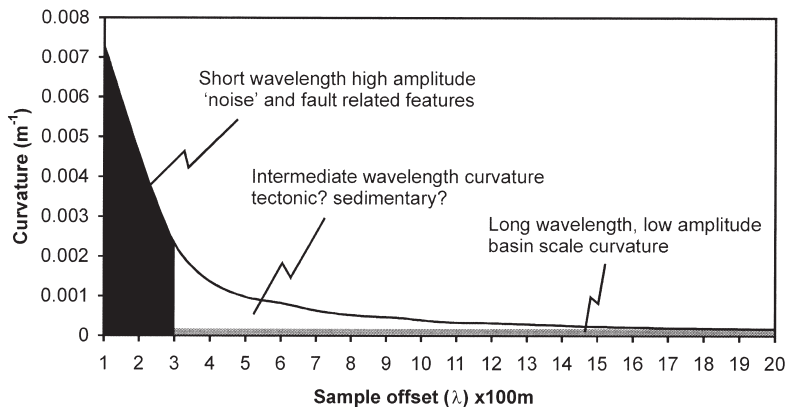


Fig. 11. Schematic diagram of how the integrals are distributed under a $K_1 + K_2$ curve. The noise and basin curvature integrals are calculated and then removed from the raw integral to leave the area associated with intermediate sample offset curvatures. The integrals are calculated using the trapezium rule.

does not require a priori knowledge of the offsets of a tectonic wavelength or wavelengths affecting the surface. However, the influence of tectonic curvatures at the same wavelengths as any short-wavelength noise will be lost in the filtering procedure. It may be more fruitful to calculate the integrals over shorter sections than has been done here. A subset of the spectra or the average spectra of a dataset could be examined to identify regions of the curves that have broadly similar characteristics. For example, based on the average spectra for the Fife Field used in Figure 11, the spectra could be subdivided into domains at 100–300 m, 300–1000 m and 1000–2000 m sample offsets.

Maps of calculated variables

The various spectra discussed above can also be used to identify an offset or groups of offsets with features of interest that can be plotted on a map or contoured surface to assess their areal distributions. For the Fife dataset, the variables plotted as part of this study were: $K_1 + K_2$, Gaussian curvature, and the integrals of the $K_1 + K_2$ spectra both with and without filters applied for noise and basin-scale curvature. For the Valhall dataset the variable plotted was $K_1 + K_2$.

$K_1 + K_2$ maps from the Fife Field

Figure 12 shows the variations in $K_1 + K_2$ for a subset of offsets for the Top Ekofisk surface of the Fife Field. Figure 12a shows the curvature variations for sample offsets of 100 m. The main features visible are broadly north–south trending linear zones of higher curvature in the west part of the dataset. These ridges of curvature are probably related to geological features that represent the effect on the cover rocks of the reactivation of faults in the deeper strata. Also visible throughout the dataset are aligned ‘nodes’ of low and high curvature values. It is possible that the isolated nodes are an artefact of the contouring process; alternatively these nodes could represent the areal distribution of the noise identified from the spectral plots at offsets equal to or less than 300 m.

The 1000 m offset curvatures are shown in Figure 12b where it can be seen that the isolated nodes visible on Figure 12a are less prominent and the higher values of curvature are distributed in broader ‘patches’. These may be related to the 500 m to 1000 m scale features visible on the topographic map. The most obvious feature at this sample offset is the small NE–SW trending closure in the west part of the dataset.

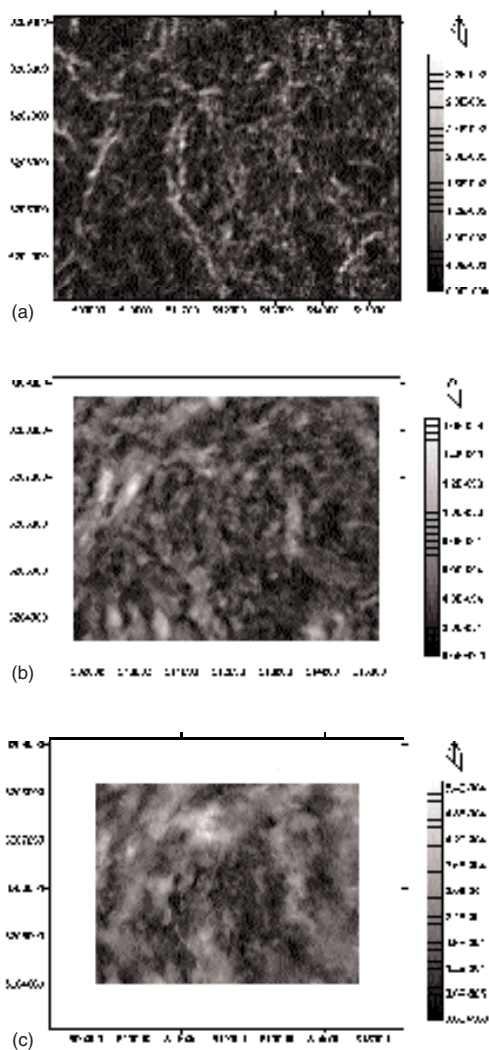


Fig. 12. Maps of $K_1 + K_2$ for the Fife Field Top Ekofisk surface at different offsets: (a) 100 m sample offset; (b) 1000 m sample offset; (c) 2000 m sample offset (XY scale in m; curvature values in m^{-1}).

The 2000 m offset curvatures seen in Figure 12c show an even wider distribution of patches of curvature than those visible in Figure 12b, although the north–south trending ridge of curvature visible on the west side of the dome in Figure 12a is also visible on this plot. The most obvious feature is the patch of higher curvature values over the crest of the dome.

It can clearly be seen that in general, the areal distributions of curvature generally become more broadly distributed with increasing sample

wavelength. This can be interpreted to be a result of the removal of short-wavelength features at increasing sample wavelengths. This has the effect of revealing more subtle features at larger sample offsets. If used in conjunction with the spectral curvature plots, these maps can help determine the range of scales over which a given feature is influencing curvature.

$K_1 + K_2$ maps from the Valhall Field

Figure 13 shows the $K_1 + K_2$ variations from the Top Tor surface of the Valhall Field. The 100 m sample offset curvatures shown in Figure 13a clearly isolate the faulting on the crest of the main pericline. Very little curvature is seen between the faults on the crest or on the flanks of the dome indicating that the curvature at short wavelengths is predominantly related to faulting rather than more dispersed features such as acquisition-related noise. However, there are some linear zones of curvature parallel to grid lines E529000 and N6230000. These are almost certainly related to the acquisition or processing or interpretation of the data rather than to any geological features.

Figure 13b shows the 1000 m sample offset curvatures. The fault-related curvatures are still obvious but affect larger areas around the faults. This could be related to fault related drag or rollover; however, in this case it is more likely to be a result of aliasing of the fault-related curvatures (throws) immediately adjacent to the faults. Also visible are some patches of curvature associated with the margins of the larger-scale pericline feature (e.g. E520000, N6239000). The map in Figure 13c shows the 1900 m sample offset curvatures. This map is dominated by the larger distributions of curvature associated with the main pericline structure. However, even at these offsets there are obvious fault related curvatures in discrete zones within the crestal region although they are less marked than in the other maps.

Fault-related, high curvature values are visible at E522500, N6235500 on all three $K_1 + K_2$ maps. The persistence of this feature over a large range of sample offsets highlights the need for subdividing the curvature map into subdomains to remove the effects of faults with throws large enough to be seen across many sample offsets. However, for the smaller throw faults it can also be seen that, as the sample offset is increased, their influence on curvature is decreased.

Fife Field Gaussian curvature maps

Figure 14 shows the variations in Gaussian curvature with increasing offset for the Top Ekofisk

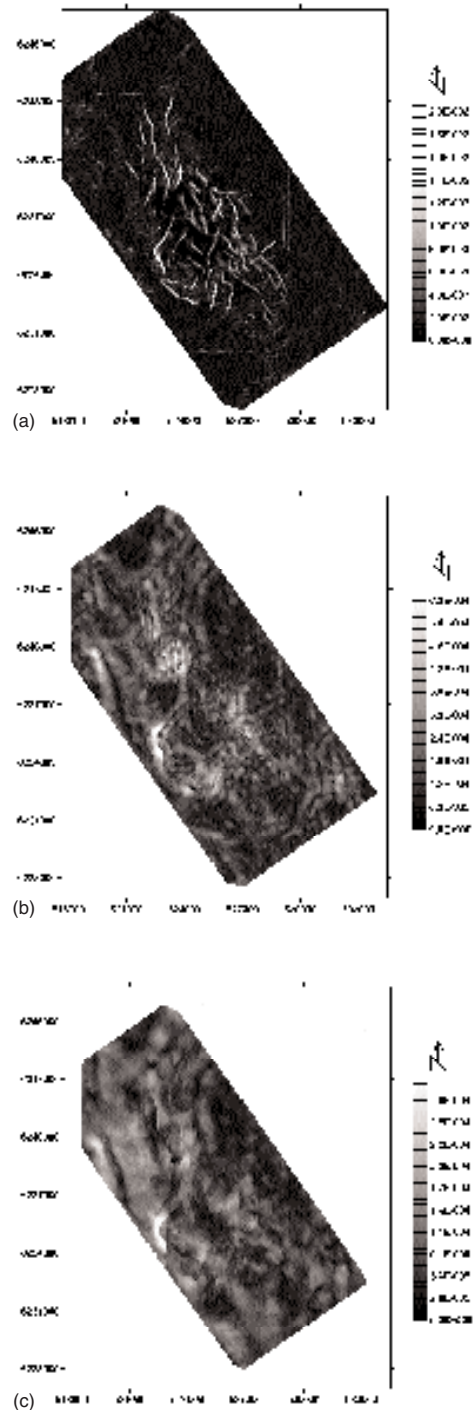


Fig. 13. Maps of $K_1 + K_2$ for the Valhall Field Top Tor surface at different offsets: (a) 100 m sample offset; (b) 1000 m sample offset; (c) 1900 m sample offset (XY scale in m; curvature values in m^{-1}).

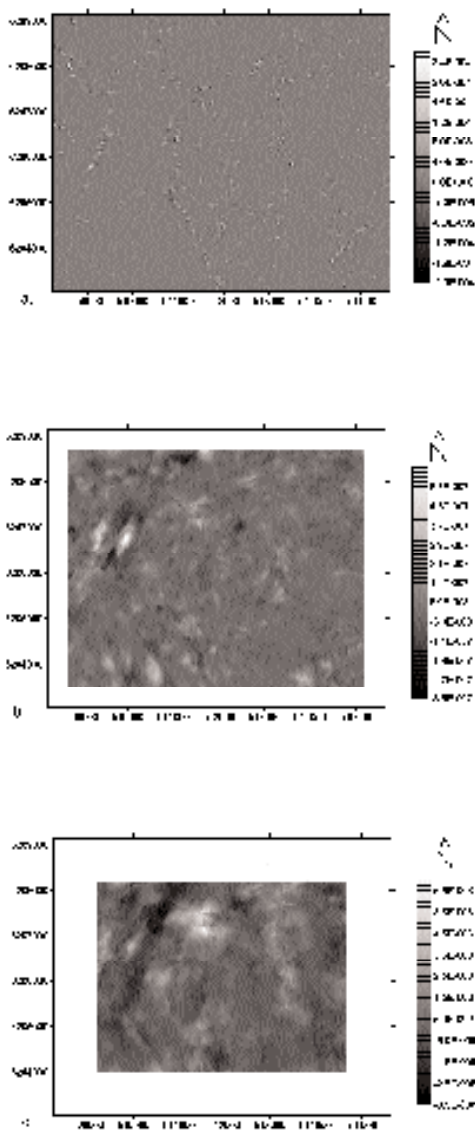


Fig. 14. Maps of Gaussian curvature for the Fife Field Top Ekofisk surface at different offsets: (a) 100 m sample offset; (b) 1000 m sample offset; (c) 2000 m sample offset (XY scale in m; curvature values in m^{-2}).

surface of the Fife Field. The spatial variability at offsets of 100 m (Fig. 14a) is markedly less than for the K_1+K_2 maps although some small, aligned ‘hot spots’ are visible on the west side of the main closure and on the smaller closure to the west. At intermediate sample offsets of 1000 m (Fig. 14b) the most obvious feature is a NNE trending dome in the west part of the

dataset. This feature is not very clear at the 100 m and 2000 m sample offsets. No alignment of features is visible to the east of the main dome at sample offsets of 100 m and 1000 m. However, at sample offsets of 2000 m (Fig. 14c) a 1 km wide, NNW trending zone of curvature is visible in the east part of the dataset.

In general, the Gaussian curvature maps appear to provide a clearer distribution of the main features on the Fife surface when compared to the K_1+K_2 maps. This is perhaps due to the suppression of grid nodes where the K_2 curvatures are very low, irrespective of the value of the K_1 curvatures. On the K_1+K_2 maps these nodes will show up if the K_1 values are high.

K_1+K_2 spectral integral maps

Figure 15 shows maps of the integrals of the spectra of K_1+K_2 for each grid node of the Top Ekofisk surface of the Fife Field. Figure 15a shows the raw integral and Figure 15b shows the map of the integral filtered for noise-related curvatures at offsets below 300 m and basin-scale curvatures at offsets above 1000 m. It can be seen that there is very little difference between Figure 15a and Figure 15b although the filtered map appears to show a smoother distribution of curvature. Any differences are probably due to the removal of the short-wavelength noise-related integrals. The effects of the removal of longer-wavelength curvatures are probably not visible in Figure 15b. This is because the values of curvature are consistently very small above offsets of about 1000 m so the larger offset curvatures will have a negligible input to the integral. It can also be seen on both plots that in general the variability in curvature across the surface is low but that there are a few ‘hot spots’ distributed throughout the field.

The maps demonstrate two features of using the integral of the spectra. Firstly, the effects of noise can be reduced so that the regularly spaced ‘nodes’ visible on the 100 m sample offset map of K_1+K_2 in Figure 12a are not present. Secondly, the integral map may allow the assessment of the distributions of the most important wavelength or wavelengths of curvature without a priori knowledge of those wavelengths.

Discussion

Spectral curvature analysis

From the various plots and maps presented above, it can be seen that curvature mapped at different grid offsets can have markedly different distributions across the mapped surface. The K_1+K_2 spectra derived from a synthetic plane

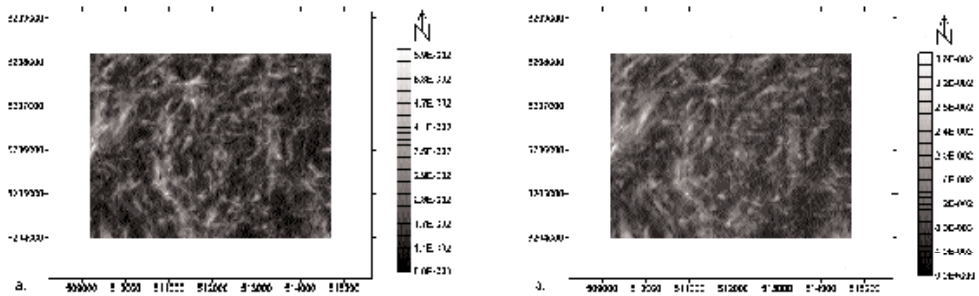


Fig. 15. Maps of the $K_1 + K_2$ spectra integrals from the Fife Field Top Ekofisk surface: (a) Raw integral without removing the short-wavelength curvatures (<300 m sample offset) or basin scale curvatures; (b) Residual integral after removal of the integrals associated with short wavelengths and basin-scale curvatures (XY scale in metres, integral values in m^{-1}).

with noise, at the grid spacing scale, showed some similar characteristics to the spectra derived from the North Sea chalk surfaces. However, the spectra derived from a plane with noise forms an extreme end member of the range of possible spectral types that could be expected from natural surfaces. In this example the power law exponent was approximately -2 . At the other extreme would be a smooth, cylindrically folded surface (see Fig. 1a) with a large radius of curvature compared to the grid spacing (e.g. 2000 m–4000 m for a 50 m grid). Such a surface would show $K_1 + K_2$ trendline power law exponents of zero because the measured curvature is constant at all scales. The trendline power law exponents of the spectra examined from the Fife and Valhall fields were typically in the range of -0.5 to -1.8 and fall within the range of values defined by the extreme examples described above. It is possible that the power law characteristics of spectra for many, if not all, natural surfaces will fall between these two extreme end member values.

Of more practical use is the fact that the examples of spectra from the Fife and Valhall fields could be related to the topography of the surfaces. Although curvature spectra cannot be used in isolation to identify the origins of features on a surface, they can be used to identify the range of sample offsets over which a particular feature influences curvature. For the Fife Field dataset, it was proposed that seismic acquisition- or processing-related noise dominated the curvature spectra at sample offsets of less than 300 m. This cut-off value could be used as a basis for smoothing the dataset prior to future studies involving curvature or other parameters. Conversely, for the Valhall Field data it was apparent that fault-related curvatures dominated

the spectra at sample offsets of less than 300 m. In this case smoothing would not improve the dataset and would probably result in loss of information with respect to the faulting. In general, it is recommended that curvature analysis is initially performed on unsmoothed data so that the full range of curvature characteristics can be established. If smoothing is performed prior to curvature analysis then some potentially useful information may be lost without a proper evaluation of the noise-to-signal ratio.

Curvature and strain

As discussed above, the principal reason for mapping the curvature of geological surfaces is to assess the distribution of curvature-related strain affecting a lithological unit or group of units. However, it will have become apparent that care needs to be taken when interpreting curvature variations. In addition to the short-wavelength curvatures produced from the seismic acquisition- and processing-related noise, there will also be some curvature from the pre-deformation topography of the surface. It will not always be possible to deconvolve this curvature from the tectonic curvature purely by examination of the curvature spectra or maps. However, it may be possible to compensate for these curvatures in a strain restoration package prior to calculating the curvature of the surface. Another approach would be to calculate the curvature variables and spectra on different realizations of a strain model. The realizations could be separated either by time or by the strain path.

Curvature and fracture permeability

There are several factors that contribute to the development of fracturing on a fold that can be

linked to permeability; only one of these factors is the present-day curvature. The strain path followed by the rock is important as it is possible that the measured curvature of a fold or a portion of a fold is not the maximum curvature that has affected the rock during the deformation history. This will be relevant to fault bend folds formed in overthrust belts where the hanging wall rocks may be 'fed' through fold hinges and then unfolded to some degree as transport progresses (cf. Ramsay & Huber 1987; Price & Cosgrove 1990). Another factor is the timing of the deformation with respect to the diagenetic history of the rocks. Deformation whilst the rocks are undercompacted may result in a more ductile mechanism for strain accommodation and the presence of considerable quantities of formation fluids may result in mineral infills of any brittle fractures that do form. The mineralization of fractures may occur at any time during the history of a structure and could cause a significant reduction in fracture permeability.

In addition to the various curvature parameters studied here, it is also possible to plot the principal curvature directions on a map. If curvature variability can be successfully linked to fracture intensity then this may provide a means to map fracture trends across a surface. If a correlation can also be made between fracture intensity and permeability then the principal curvature orientations could be used for mapping permeability trends across the surface. For this to be successful the relationship of the fracture sets to the fold geometries needs to be known or estimated. The simplest case is for fracturing to strike perpendicular to K_1 as in tensile fractures formed in the outer arcs of tangential longitudinal strain folds. It may be possible to determine other fold geometry to fracture orientation relationships for other tensile fractures or shear fracture sets. The use of the spectral curvature plots and maps in this process would allow an assessment of the scales for which the correlation holds.

As part of a comprehensive study of the deformation style and history of a structure, spectral curvature mapping is a potentially useful tool in identifying regions of higher fracture density. It should also be noted that a comprehensive assessment of the fracture distributions within a reservoir unit or units will require the input of fracture data gathered from core or wellbore imagery. These data will allow the identification of the lithomechanical units and possibly the folding styles. These fracture data can also be used to test and calibrate any models of fracture distribution and density derived from curvature variations.

Conclusions

The spectral curvature mapping techniques described and discussed in this paper are particularly useful in allowing the analysis of curvatures at a variety of wavelengths affecting a point or group of points on a gridded surface.

A variety of variables can be plotted. Spectra of the sum of the absolute curvatures ($K_1 + K_2$) and Gaussian curvature allow the identification of curvatures at a variety of different wavelengths.

The representation of several different wavelengths of curvature can be achieved by using the integral of the $K_1 + K_2$ versus offset plot. This integral can be filtered for short-wavelength noise or features and long-wavelength basin-scale curvatures.

Maps of various curvature types can be used to assess the changing areal distribution of curvature with increasing offset. Particularly useful variables are $K_1 + K_2$ and Gaussian curvature.

Maps of the integral of the $K_1 + K_2$ variable indicate that it can show the combined distribution of curvature from several different wavelengths.

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